AIAA JOURNAL
Vol. 36, No. 9, September 1998

Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Turbulence Modeling for Time-Dependent RANS and VLES: A Review"

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IN Ref. 1, Speziale has proposed new subgrid scale models that allow a direct numerical simulation (DNS) to go continuously to a Reynolds averaged Navier–Stokes computation (RANS) through large eddy simulations (LES) and very large eddy simulations (VLES), depending on the level of resolution. Two points are fundamental in his approach. The first is the parameterization of the resolution level in terms of the ratio r between Δ, the computational mesh size, and L_K, the Kolmogorov length scale,

$$r = \Delta / L_K \tag{1}$$

and the second is to recover a state-of-the-art Reynolds stress model in the limit of $r \to \infty$, so that an LES goes continuously to a RANS computation. As stated on page 179 of his paper [Eq. (78)], this can be accomplished in mathematical terms by models of the form

$$\tau_{ij} = [1 - \exp(-\beta r)]^n \tau_{ij}^{(R)} \tag{2}$$

where τ_{ij} is the subgrid-scale stress model, $\tau_{ij}^{(R)}$ is a Reynolds stress model, and β and n are constants.

In our opinion some arbitrariness of this very interesting approach could be removed by applying the dynamic modeling procedure. Let us rewrite Eq. (2) in the following general form:

$$\tau_{ij} = f(r)\tau_{ij}^{(R)} \tag{3}$$

and let us argue which is the best approximation to the function f. We know that the Reynolds stress model and the subgrid-scale stress model must be related by the identity[2]

$$\tau_{ij}^{(R)} = \langle \tau_{ij} \rangle + T_{ij} \tag{4}$$

where $\langle \cdots \rangle$ is the infinite time average and T_{ij} is the resolved stress given by

$$T_{ij} = \langle \bar{u}_i \bar{u}_j \rangle - \langle \bar{u}_i \rangle \langle \bar{u}_j \rangle \tag{5}$$

The identity (4) can be easily verified by noting that the Reynolds stress $\tau_{ij}^{(R)}$ and the subgrid-scale stress are given formally by the relations

$$\begin{aligned} \tau_{ij}^{(R)} &= \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle \\ \tau_{ij} &= \overline{u_i u_j} - \bar{u}_i \bar{u}_j \end{aligned} \tag{6}$$

where the overbar stands for the partial average at the particular resolution level and where we have assumed that

$$\langle \overline{\cdots} \rangle = \langle \cdots \rangle \tag{7}$$

This last condition is in our opinion as essential for a practical LES as the conditions examined by Speziale in his paper,[1] and we refer to Ref. 3 for more details on that.

Let us now explain how the identity (4) can be dynamically exploited to produce a function $f(r)$ consistent with the results. We notice that the resolved stress T_{ij} can be calculated during the simulation with an increasing accuracy, and if we substitute in Eq. (4) expression (3) for τ_{ij}, the residual

$$Q_{ij} = \tau_{ij}^{(R)} - f(r)\tau_{ij}^{(R)} - T_{ij} \tag{8}$$

should be minimized in the least-square sense. This procedure is similar to that applied by Lilly[4] to the dynamic model,[5] and the final result is

$$f(r) = 1 - \frac{\tau_{ij}^{(R)} T_{ij}}{\tau_{ij}^{(R)} \tau_{ij}^{(R)}} \tag{9}$$

that produces a function $f(r)$ that correctly goes to one when $T_{ij} \to 0$, the RANS limit, and goes to zero when $T_{ij} \to \tau_{ij}^{(R)}$, the DNS limit. In conclusion we think that the approach proposed by Speziale is extremely interesting from both a practical and a theoretical point of view. As remarked in his paper, the state of the art of the existing Reynolds stress models and their applicability to nonequilibrium turbulent flows makes their extension to LES very attractive. The capability of a model of bridging spontaneously the gap between DNS, LES, and RANS is a fundamental issue, and the proposal of a modulating function very stimulating. A dynamic approach to the determination of this bridging function could be worthwhile to explore.

References

[1]Speziale, C. G., "Turbulence Modeling for Time-Dependent RANS and VLES: A Review," *AIAA Journal*, Vol. 36, No. 2, 1998, pp. 173–184.

[2]Germano, M., "Turbulence: The Filtering Approach," *Journal of Fluid Mechanics*, Vol. 238, 1992, pp. 325–336.

[3]Germano, M., "A Statistical Formulation of the Dynamic Model," *Physics of Fluids*, Vol. 8, No. 2, 1996, pp. 565–570.

[4]Lilly, D. K., "A Proposed Modification of the Germano Subgrid Scale Closure Method," *Physics of Fluids A*, Vol. 4, No. 3, 1992, pp. 633–635.

[5]Germano, M., Piomelli, U., Moin, P., and Cabot, W. H., "A Dynamic Subgrid-Scale Eddy Viscosity Model," *Physics of Fluids A*, Vol. 3, No. 7, 1991, pp. 1760–1765.

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Received April 20, 1998; accepted for publication June 14, 1998.

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Reply to M. Germano

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GERMANO presents some relevant points. Indeed, it may be possible to use the formalism of the dynamic subgrid-scale model to develop the form of the grid function that is now empirically based. I have only one concern. The formalism of the dynamic subgrid-scale model is based on a test filter. This compromises the ability to apply this technique in complex geometries. That is why I stayed away from this technique. However, it is possible to calibrate the model by this means, although I have some lingering doubts. There is no question that the correct parameter to calibrate subgrid-scale models is the ratio of the grid size to the Kolmogorov length scale, as discussed at length in the paper. This is the parameter that determines how well resolved a computation is in the numerical simulation of turbulence. (When this parameter is of order one or less, one has a direct simulation where all relevant scales are resolved.) It remains an open question as to what the specific functional dependence is in the grid function. Perhaps the dynamic subgrid-scale approach will be useful in this regard.

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Editor-in-Chief

Received May 24, 1998; accepted for publication June 15, 1998.

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